Homework 8

due Wednesday, April 5, 5pm

Submit zipped `.m` files on Canvas and printed published files in 182 George St box #15 or #16

You are encouraged to work with other students on this assignment but you are expected to write and work on your own answers. You don't need to provide the name of students you worked with. You can find information about usage and syntax of any built-in Matlab function by typing

`help␣`⟨function name⟩

in the Command window, where ␣ is the space character.

You are expected to submit

(I) a zipped file containing all your .m and any `.out` files online on Canvas

(II) a printout of the files produced by the `publish` command of the filled in template and all other .m files your template uses in the dropbox in 182 George St lobby (#15 for S01 and #16 for S02)

Late homework will not be accepted without proper justification.

Problem 1 : Newton-Cotes rules

For this problem, you will find the m-piont Newton-Cotes rules for $2 \leqslant m \leqslant 8$, and test the accuracy of the numerical integration. You shall consult lecture note `Lecture 21` to finish this homework.

(a) Write a Matlab function with the definition

```
function w = NCweights(m)
```

where

`m` : number of quadrature points.
`w` : a column vector of length m,
storing the quadrature weights of m-point Newton-Cotes rule on the unit interval.

The weights for $m = 2, 3, 4$ are already provided in the note `Lecture 21`, you shall continue to obtain the weights, which are rational numbers, for $m = 5, 6, 7, 8$. *Hint: you might want to use the code* `lagrangeIntegral.m` *provided in Canvas to compute the weights.*

(b) Write a Matlab function with the definition

```
function numI = QNC(f,a,b, m)
```

to compute the m-point Newton-Cotes quadrature rule for the integral $\int_a^b f(x)dx$.

(c) Use your function `QNC.m` to compute the integral of $f(x) = e^{-x^2/2}$ on the interval $[-3, 3]$ with m-point Newton-Cotes rule for $m = 2, 3, 4, 5, 6, 7, 8$. The exact integral is $\sqrt{2\pi}\,\mathrm{erf}(3\sqrt{2}/2)$, where erf is a matlab built-in function. Print out the numerical integral along with the absolute error in a table. *Hint: consult code* `testQNC.m` provided in Canvas on printing out a formatted table on command window.

Problem 2 : composite (open) trapezoidal method

For this problem, you will compute the integral of a function using the composite (open) trapezoidal method, see lecture note `Lecture 20` for its definition on a single interval.

(a) Write a Matlab function that compute the numerical integral using the composite open trapezoidal method with the definition

```
function numI = compOpenTrapez(f, a, b, N)
```

where

`f`	:	function to be integrated,
`a, b`	:	left and right end points of the interval,
`N`	:	number of intervals,
`numI`	:	returned numerical integral.

You shall derive the formulation by yourself.

(b) Use your function `compOpenTrapez.m` to compute the degree of exactness for the open trapezoidal method as follows:
Compute the numerical integral for the function $f(x) = x^m$ on the unit interval $[0, 1]$ with the open trapezoidal method on the whole interval. (*You shall fix left and right endpoints* $a = 0$, $b = 1$, *and fix the number of intervals to be one:* $N = 1$ *when you call the function* `compOpenTrapez.m`)

Start with $m = 0$

Print out the numerical integral along with its absolute error.

i. if the error is within machine precision $\approx 10^{-14}$, increase m by one, and continue to evaluate the numerical integral, and obtain the error.

ii. if the numerical error is not within machine precision, stop the code and print out the degree of exactness on screen.

(c) Use your function `compOpenTrapez.m` to compute the integral of $f(x) = e^{-x^2/2}$ on the interval $[-3, 3]$ with N uniform intervals. Take number of intervals N to be 2^k, with $k = 2, 3, 4, 5, 6, 7, 8, 9, 10$, The exact integral is $\sqrt{2\pi}\,\mathrm{erf}(3\sqrt{2}/2)$, where `erf` is a matlab built-in function. Print out the number of divided intervals along with the absolute numerical error in a table. Plot the error against the divided interval length $(b - a)/N = 6/N$ using `loglog` scale.

(d) Repeat part (c) with the provided code `compTrapez.m`(composite closed trapezoidal method) in canvas. Put the two `loglog` plots on the same figure. Do you observe similar convergence?